Multiparent Path Relinking: A metaheuristic framework

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Abstract. This work presents the general framework for Multiparent Path Relinking (MPR) proposed in Martín-Santamaría et al [5], an advanced extension of the traditional Path Relinking (PR) strategy. While PR has been extensively used for search intensification by exploring trajectories between pairs of solutions, MPR extends this concept by simultaneously exploiting information from multiple elite guiding solutions. In this work, we summarize the fundamental components of the methodology presented in Martín-Santamaría et al [5], including Basic, Weighted, and Probabilistic relinking mechanisms, as well as alternative strategies for selecting initiating and guiding solutions from an elite set. The proposed framework is illustrated through its application to the Power Dominating Set Problem (PDSP), where MPR is embedded within GRASP and Scatter Search metaheuristics. Computational results demonstrate that MPR effectively enhances search intensification, leading to improved solution quality while maintaining competitive computational effort. These findings highlight the potential of MPR as a versatile building block for the design of advanced metaheuristic algorithms.

Keywords: Multiparent Path Relinking, Scatter Search, GRASP.

1 Introduction

Path Relinking (PR) is a trajectory-based intensification strategy that has become a fundamental component of many metaheuristic frameworks. Originally introduced in the context of Tabu Search by Glover [1] and later formalized within the Scatter Search methodology by Glover et al. [2], PR generates a sequence of moves that gradually transforms an initiating solution into a guiding solution. By exploring trajectories connecting high-quality solutions, PR seeks to identify new solutions that combine desirable attributes inherited from both endpoints. Owing to its effectiveness, PR has been

successfully integrated into a wide variety of metaheuristics, including GRASP, Scatter Search, and multiobjective optimization procedures [4, 6].

Multiparent Path Relinking (MPR) extends the traditional PR paradigm by replacing the single guiding solution with a set of elite guiding solutions. Although the concept was originally proposed by Glover et al. [2], it remained largely unexplored for more than two decades. In Martín-Santamaría et al. [5], we presented the first comprehensive implementation and computational assessment of MPR, demonstrating its potential as an effective search intensification mechanism. In MPR, an initiating solution (S_i) is guided toward a set of solutions ($S_{g1}, S_{g2}, \dots, S_{gk}$), allowing the search to exploit information from multiple elite solutions. This richer information structure creates new opportunities for combining high-quality attributes and exploring promising regions of the solution space.

The main challenge in MPR lies in determining how information from multiple guiding solutions should be aggregated and exploited during the relinking process. In particular, attributes inherited from different guiding solutions must be appropriately weighted and selected to balance intensification and diversification while preserving solution feasibility. The remainder of this paper is organized as follows. Section 2 presents the main components of the MPR framework, including the alternative relinking mechanisms. Section 3 discusses its integration as a general search component within different metaheuristic paradigms. Section 4 summarizes the computational results, and Section 5 concludes the paper.

2 Multiparent Path Relinking Framework

Multiparent Path Relinking (MPR) was originally proposed by Glover et al. [2] as a generalization of classical Path Relinking in which an initiating solution is guided simultaneously by multiple elite solutions rather than by a single guiding solution. A complete implementation and computational evaluation of this concept was presented in Martín-Santamaría et al. [5], where we introduced a reference framework and demonstrated its effectiveness on the Power Dominating Set Problem. The framework operates by constructing trajectories that progressively transform an initiating solution (S_i) toward the set of attributes contained in a collection of guiding solutions ($S_{g1}, S_{g2}, \dots, S_{gk}$). The central component of MPR is the mechanism used to evaluate and incorporate attributes from the guiding solutions during the relinking process. Three alternative variants have been proposed.

In the *Basic Multiparent Path Relinking* (BMPR) variant, each attribute is assigned a score based on its relative frequency among the guiding solutions. Attributes that appear more frequently are considered more relevant and are therefore preferred during the path construction process. Relinking is performed through a sequence of insertions and removals that gradually replace low-frequency attributes in the initiating solution with highly ranked attributes from the guiding solutions.

The *Weighted Multiparent Path Relinking* (WMPR) variant extends this idea by incorporating solution quality into the attribute evaluation process. Specifically, the

contribution of each attribute is weighted according to the quality of the guiding solutions in which it appears. As a result, attributes belonging to higher-quality solutions receive greater importance, biasing the search toward more promising regions of the solution space.

Finally, the *Probabilistic Multiparent Path Relinking* (PMPR) variant introduces a stochastic component into the relinking process. Instead of considering all guiding solutions simultaneously, a guiding solution is selected probabilistically at each iteration, with selection probabilities proportional to solution quality. Consequently, the search trajectory dynamically changes during the relinking process, providing an additional source of diversification.

These three variants represent different mechanisms for balancing intensification and diversification within the MPR framework and can be embedded within a wide range of trajectory-based and population-based metaheuristics.

3 Metaheuristic integration

The proposed MPR framework can be embedded within different metaheuristic paradigms. As demonstrated in Martín-Santamaría et al. [5], MPR can operate either as an intensification mechanism in trajectory-based methods or as a combination operator in population-based approaches. To illustrate this flexibility, the framework was integrated into GRASP and Scatter Search and evaluated on the Power Dominating Set Problem (PDSP).

In a GRASP framework, MPR is applied as a post-processing intensification phase. After generating and improving a collection of elite solutions, MPR explores trajectories among these solutions to identify new high-quality regions of the search space. In contrast, within Scatter Search (SS), MPR acts as the solution-combination mechanism, generating trial solutions from a set of elite reference solutions that are subsequently improved and incorporated into the search process.

To illustrate the applicability of the framework, MPR was evaluated on the Power Dominating Set Problem (PDSP), an NP-hard graph optimization problem arising in the monitoring of electrical power networks. The objective is to identify the minimum set of monitoring devices required to observe the entire network through domination and propagation rules. The PDSP provides a challenging testbed for MPR because intermediate solutions generated during the relinking process may become infeasible, requiring repair mechanisms to restore feasibility. This characteristic makes the problem particularly suitable for assessing the robustness and effectiveness of multiparent relinking strategies.

4 Computational Experiments

The computational study conducted in Martín-Santamaría et al. [5] demonstrates the effectiveness of the MPR framework when embedded within both GRASP and Scatter Search. The performance of the proposed approaches was assessed on a benchmark set of Power Dominating Set Problem instances and compared against the Fixed Set Search

(FSS) metaheuristic of Jovanovic and Voß [3], which represents the state-of-the-art metaheuristic proposed for this problem. The results show that MPR consistently enhances the search process by exploiting information from multiple elite solutions, leading to improved solution quality over classical Path Relinking approaches. Among the different implementations, Scatter Search combined with MPR achieved the best overall performance on large-scale instances, obtaining the lowest average deviation from the best-known solutions and identifying more best-known solutions than both GRASP-based MPR and FSS. These findings indicate that MPR constitutes an effective and versatile search component that can significantly strengthen both trajectory-based and population-based metaheuristics.

5 Conclusions

Multiparent Path Relinking (MPR) extends the classical Path Relinking paradigm by simultaneously exploiting information from multiple elite solutions, providing a flexible framework for balancing intensification and diversification in metaheuristic search. Through alternative mechanisms for aggregating guiding-solution attributes and its integration within both GRASP and Scatter Search, MPR has demonstrated its ability to improve solution quality for challenging combinatorial optimization problems. The computational results reported in the literature highlight its effectiveness as a general search component and suggest that MPR represents a promising building block for the design of advanced metaheuristics. Future research should investigate its application to other optimization problems as well as its integration with adaptive memory and learning-based mechanisms.

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